From the Mathematical Tripos

§ 8. It is always pleasant to find others doing the silly things one does oneself. The following appears as a complete question in Schedule B for 1924 (Paper I).

(*a*) [1] An ellipsoid surrounded by frictionless homogeneous liquid begins to move in any direction with velocity V. Show that if the outer boundary of the liquid is a fixed confocal ellipsoid, the momentum set up in the liquid is $-MV$, where M is the mass of the liquid displaced by the ellipsoid.

[The result was later extended to other pairs of surfaces, e.g. two coaxial surfaces of revolution.]

Whatever the two surfaces are we can imagine the inner one to be filled with the same liquid; then the centre of mass does not move.

Published sets of examination questions contain (for good reasons) not what was set but what ought to have been set; a year with no correction is rare. One year a question was so impossibly wrong that we substituted a harmless dummy.

There used to be 'starred' questions in Part II (present style), easy, and not counting towards a 1st. A proposed starred question was once rejected, proposed and rejected as too hard for an unstarred one, and finally used as a question in Part III.

Once when invigilating I noticed, first that the logarithm tables provided did not give values either for e or for $\log e = \cdot 4343$, secondly that question 1 asked for a proof that

[1] Let the amateur read bravely on.

something had the numerical value 4·343 (being in fact 10 log e). Was I to announce the missing information, thereby giving a lead ? After hesitation I did so, and by oversight committed the injustice of not transmitting the information to the women candidates, who sat elsewhere.

§ 9. I inherited Rouse Ball's 'Examiner's books' for the Triposes of round about 1881. In passing, some details may be of interest. The examination was taken in January of the 4th year. In one year full marks were 33,541, the Senior Wrangler got 16,368, the second 13,188, the last Wrangler 3123, the Wooden Spoon (number ninety odd) 247. The first question carried $6+15$ marks, the last question of the 2nd-4-days problem paper 325 (>247).

As a staunch opponent of the old Tripos I was slightly disconcerted to find a strong vein of respectability running throughout. It is surprising to discover that a man who did all the bookwork (which was much the same as it is now) and nothing else would have been about 23rd Wrangler out of 30. Since even the examiners of the '80's sometimes yielded to the temptation to set a straightforward application of the bookwork as a rider, he would pick up some extra marks, which we may suppose would balance occasional lapses. The two heavily marked problem papers contained of course no bookwork for him to do ; if we suppose that he scored a quarter of the marks of the Senior for these papers, or say 7 per cent. of the total, he would go up to about 20th. (Round about 1905 the figures would be 14th Wrangler out of 26 for pure bookwork, rising to 11th on 7 per cent. of the problem papers, and incidentally straddling J. M. Keynes.)

§ 10. On looking through the questions, and especially the problem papers, for high virtuosity (preferably vicious) I was again rather disappointed ; two questions, however, have stuck in my memory.

(*b*) A sphere spinning in equilibrium on top of a rough horizontal cylinder is slightly disturbed; prove that the track of the point of contact is initially a helix.

* Pursuing this idea an examiner in the following year produced (Jan. 18, morning, 1881, my wording).

(*c*) If the sphere has a centrally symmetrical law of density such as to make the radius of gyration a certain fraction of the radius then, whatever the spin, the track is a helix so long as contact lasts. [Marked at 200; a second part about further details carried another 105.]*

The question about (*b*) is whether it can, like (*a*), be debunked. On a walk shortly after coming across (*b*) and (*c*) I sat down on a tree trunk near Madingley for a rest. Some process of association called up question (*b*), and the following train of thought flashed through my mind. 'Initially a helix' means that the curvature and the torsion are stationary at the highest point P; continue the track backwards; there is skew-symmetry about P, hence the curvature and torsion are stationary. I now ask: is this a proof, or the basis of one, how many marks should I get, and how long do you take to decide the point?

* § 11. Proceed to (*c*). I do not regard this question [1] as vicious: it involves the general principles of moving axes with geometrical conditions; a queer coincidence makes the final equations soluble, but this is easy to spot with the result given. The extremely elegant result seems little known.

Take moving axes at the centre of the sphere with Oy along the normal to the point of contact, making an angle, θ say, with the vertical, and Oz parallel to the cylinder.

[1] The actual question gave the law of density and left the radius of gyration to be calculated, and asked for some further details of the motion.

Eliminating the reactions at the point of contact we get (cp. Lamb, *Higher Mechanics*, 165-166):

$$(I+Ma^2)b\ddot{\theta}=Ma^2g\sin\theta,$$
$$(I+Ma^2)\dot{w}=Ia\,\dot{\theta}q,$$
$$a\dot{q}+w\dot{\theta}=0,$$

which, on normalizing to $a=1$, become respectively, say,

$$\lambda\ddot{\theta}=\sin\theta,\ \mu^2\dot{w}=\dot{\theta}q,\ \dot{q}=-w\dot{\theta}.$$

The 2nd and 3rd of these lead to,

$$q=n\cos(\theta/\mu),\ \mu w=n\sin(\theta/\mu),$$

where n is the initial spin. If $\mu=2$ these combine with the first to give $w=\frac{1}{4}n\lambda\theta$ and so $z=\frac{1}{4}n\lambda\theta$.*

Suppose a sphere is started rolling on the inside of a rough *vertical* cylinder (gravity acting, but no dissipative forces); what happens? The only sensible guess is a spiral descent of increasing steepness; actually the sphere moves up and down between two fixed horizontal planes. Golfers are not so unlucky as they think.

Some time about 1911 an examiner A proposed the question: E and W are partners at Bridge; suppose E, with no ace, is given the information that W holds an ace, what is the probability p that he holds 2 at least? A colleague B, checking A's result, got a different answer, q. On analysis it appeared that B calculated the probability, q, that W has 2 aces at least given that he has the spade ace. p and q are not the same, and $q>p$.

Subject always to E's holding no ace, $1-q$ is the probability of W holding S ace only, divided by the probability of his holding at least S ace; $1-p$ is the probability of his holding 1 ace only, divided by the probability of his holding at least 1 ace. The 2nd numerator is 4 times the 1st. Hence

$$\frac{1-p}{1-q}=\frac{(\text{probability of S ace at least})+\cdots+(\text{probability of C ace at least})}{(\text{probability of 1 ace at least})}$$

Since the contingencies in the numerator overlap, this ratio is greater than 1.

The fallacy '$p=q$' arises by the argument: 'W has an ace; we may suppose it is the spade'. But there is no such thing as 'it'; if W has more than one the informer has to *choose* one to be 'it'. The situation becomes clearer when a hand of 2 cards is dealt from the 3 cards, S ace, H ace, C 2. Here we know in any case that the hand has an ace, and the probability of 2 aces is $\frac{1}{3}$. If we know the hand has the S ace the probability of 2 aces is $\frac{1}{2}$.